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The President communicated observations by Mr. S. Horton of Encke's Comet, and of the planet *Saturn*, extracted from the Hartwell Observatory Register. The comet was seen:—

	1861, Nov. 24.	Nov. 26.	Nov. 30.
Right Ascension	$22^h\ 31^m$	$22^h\ 29^m$	$22^h\ 25^m$
Declination	$7^\circ\ 53'$	$7^\circ\ 29'$	$6^\circ\ 49'$
Hour Angle	$2^h\ 13^m$	4^h	2^h
Clock	$0^h\ 45^m$	$2^h\ 31^m$	$0^h\ 26^m$

Power 22 and 5c.

Among the presents to the Society was included a volume of 111 Indian-ink drawings, by Mr. S. Gorton, of the planet *Jupiter*, radius about one inch, from observations made at Downs Road, Clapton, in the years 1859–60–61. The author remarks that as the observations were taken for the purpose of recording any great or marked changes in the principal belts, with an instrument of moderate size, the representation of those minute details for which great power was necessary was not attempted. The earlier drawings were made under some disadvantages—the telescope being used at a window, and not having an equatoreal mounting; this was added in April 1860, after which the planet was observed more regularly.

The power used was almost invariably 100; the telescope by Ross, 4 feet focal length, $3\frac{3}{8}$ inches aperture.

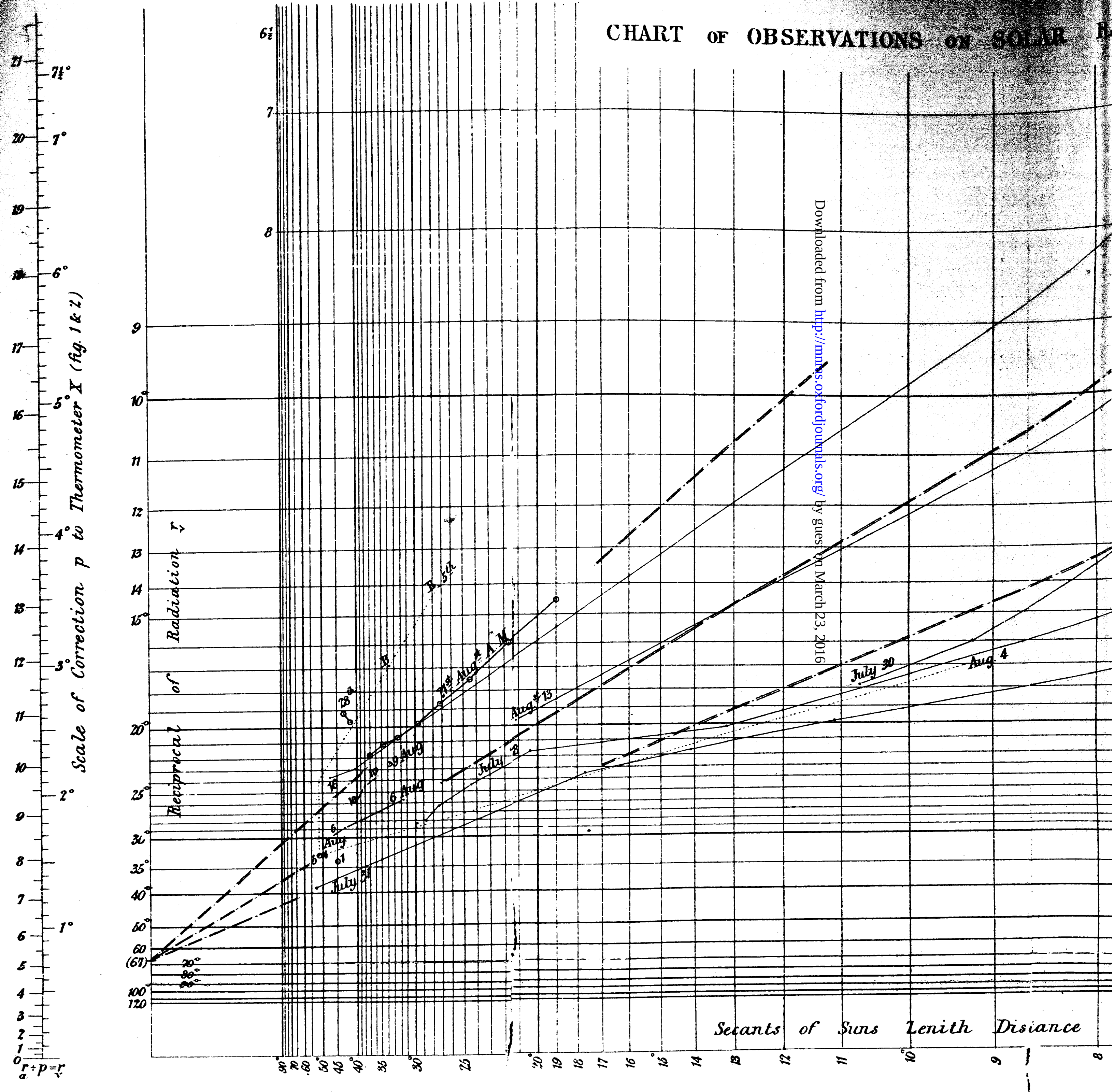
An Account of Observations on Solar Radiation.

By John James Waterston, Esq.

§ 1. In March of last year I submitted to the Society some computations with reference to the Sun's heat, and suggesting a mode of deducing the potential temperature of its radiating surface. This last summer I have endeavoured to put this method to the proof by a series of observations on solar radiation, supplemented with experiments on the rate of cooling of thermometers in air and in vacuo with different kinds of radiating surfaces; also by another series applying the method of deducing the potential temperature of a radiating surface to predicate the temperature of one that is maintained at a constant *known* temperature.

The success of these last mentioned, so far as they have as yet been carried, encourages me to lay before the Society the observations on solar radiation, with an account of the method

CHART OF OBSERVATIONS ON SOLAR R



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AR RADIATION

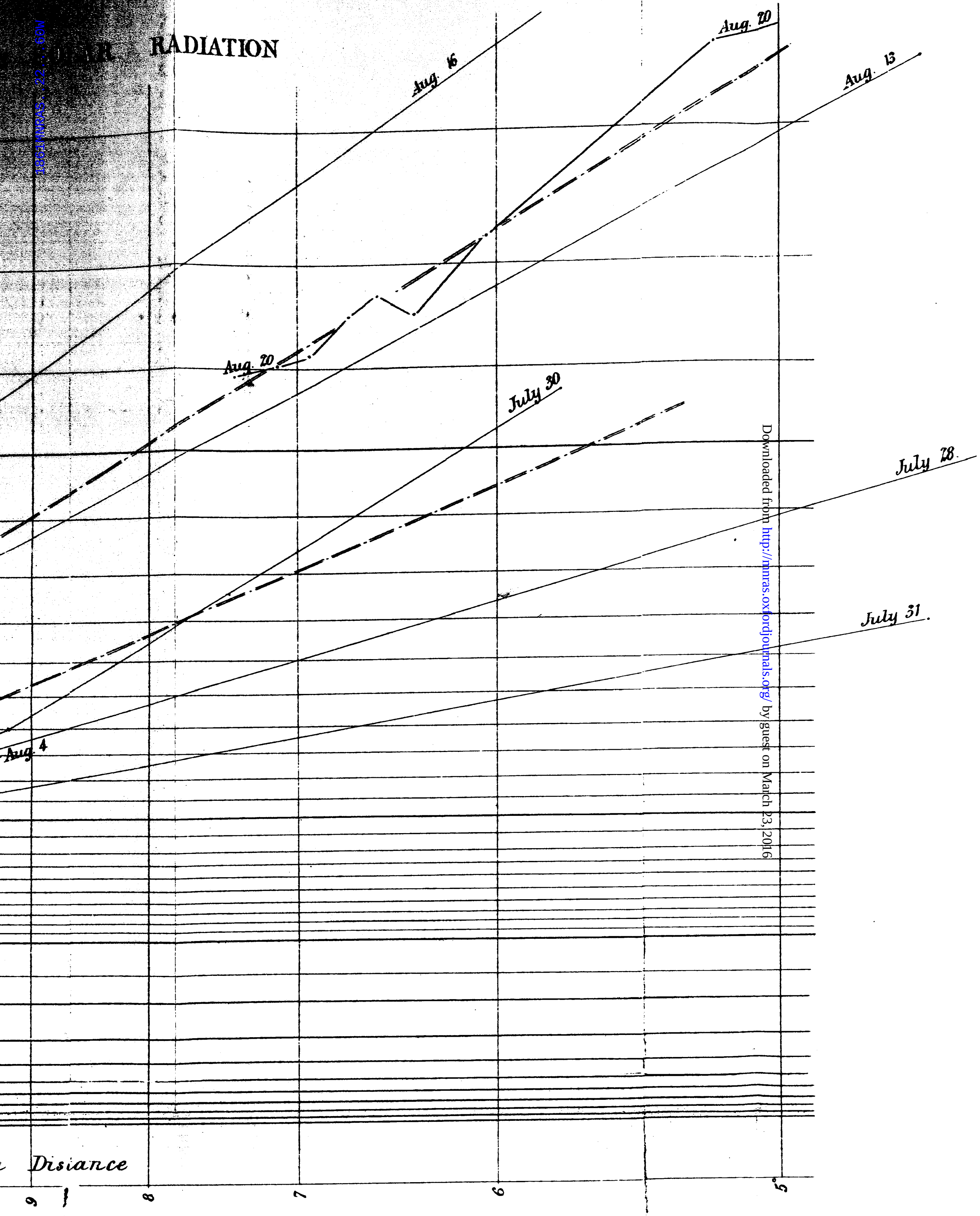

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Fig. 1.

(Scale 1 in. = 2 in.)

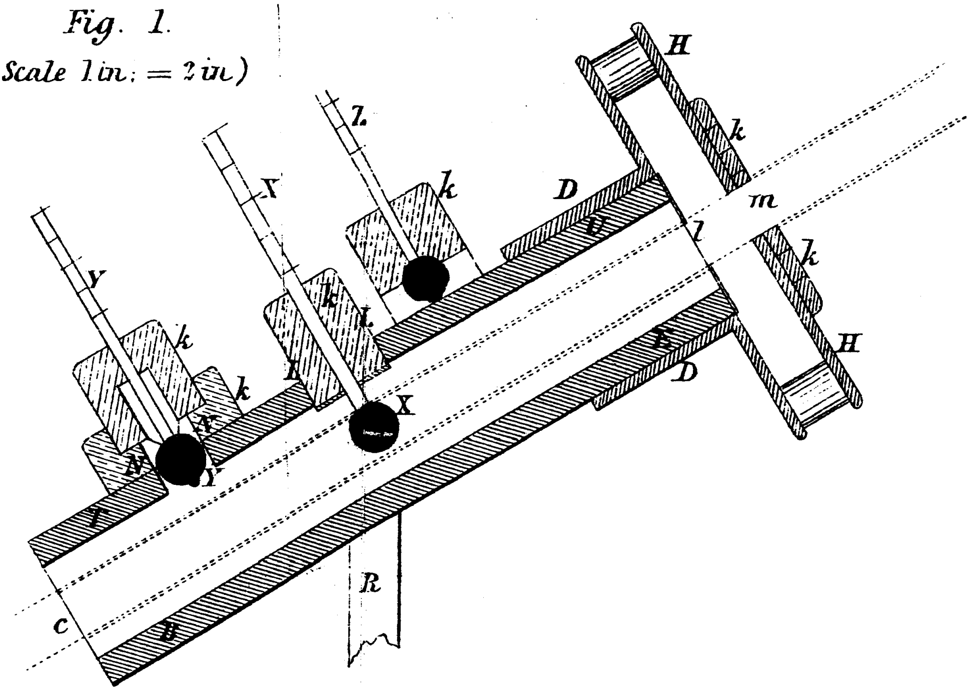
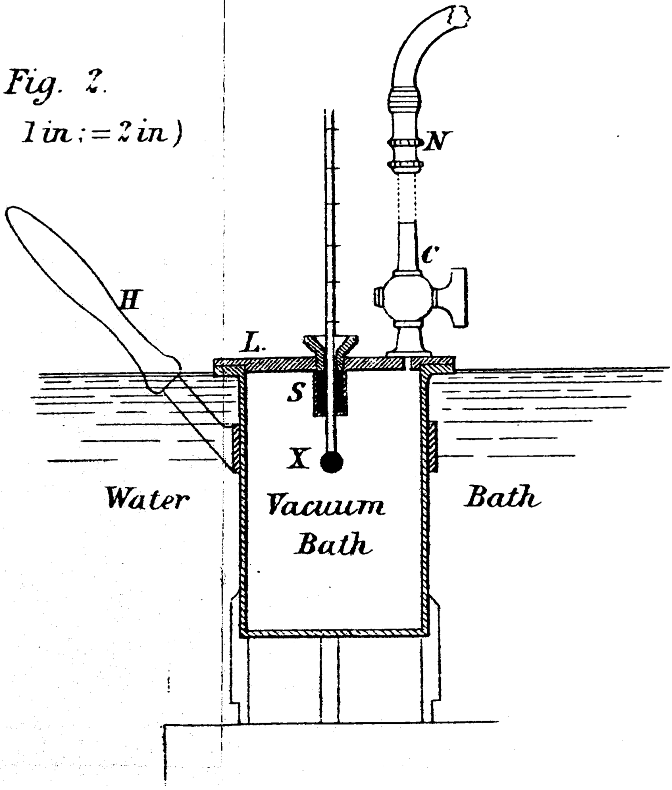


Fig. 2.

(Scale 1 in. = 2 in.)



Malby & Sons Lith.

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employed to obtain the results, and of the mode of reducing them to a vacuum.

§ 2. It will be remarked, on inspecting the chart in which the observations are projected, that a simple law of atmospheric absorption is indicated, which, if confirmed by other similar observations in different climates, would perhaps lead to more exact ideas of the influence of the atmosphere on the Sun's rays. Unfortunately the best part of the summer had passed before I could begin to observe, and there was almost constant interruption with clouds and unsettled weather. In a tropical station, where the Sun rules in a cloudless sky, the presumed law might soon be put to the test, and the heating power of the Sun's rays before entering the atmosphere ascertained with precision. Having determined this for the Earth's mean distance from the Sun, its value for any other planetary distance may be deduced by the law of the inverse square.

§ 3. When a thermometer is exposed to the Sun with its bulb blackened, it is presumed to absorb all the heat that impinges on a plane surface equal to the transverse section of the bulb; it rises and is maintained at a certain temperature; and when this balanced condition is attained we can with certainty assert that the amount that issues from the bulb is precisely equal to the quantity that enters. The elevation of its temperature above surrounding bodies due to the Sun's radiant power (which is denoted by the symbol r) would be an exact measure of that power, if no heat issued from it except by radiation, and if the rate at which heat was emitted from it increased exactly in proportion to r. Now I find that if the bulb of a thermometer is enclosed in a vacuum, the walls of which are brass, coated with lamp-black, the rate at which it cools is *exactly* proportional to the value of r, and this rate has exactly the same value, whether the glass bulb is uncoated or coated with lamp-black. When enclosed in air the rate of cooling increases faster than r. The mode of measuring the rate in both cases, and of reducing the values of r observed in air to what they would be in a vacuum, are described at the end of this paper.

§ 4. The instrument employed was designed so that the thermometer exposed to the Sun's rays should radiate against an enclosing metallic surface coated with lamp-black, and so that the temperature of that surface should always be known.

Fig. 1, with the description that accompanies it, gives the details. The rays of the Sun were admitted to strike upon the bulb of the thermometer, X, through a hole but little larger than its diameter, and were entirely screened off the brass tube against the blackened inner surface of which the radiation of the bulb took place. The thickness of the sides of the brass tube was $\frac{1}{4}$ inch, and the thermometer, Y, that indicated its temperature, was lodged in a hole cut in the upper side. The circumference of the bulb touched the brass, and its upper side

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was enclosed with cork, while the lower was exposed to the air within the tube, but was untouched by the rays of the Sun that passed through. The internal diameter of the tube was 0·9 inch and length 6 inches. The bulb of the solar thermometer was 0·42 inch in diameter, spherical in shape, and fixed in the centre, as shown in the figure. Its shadow was an easy guide in moving the tube in altitude and azimuth to keep pace with the Sun. It is difficult, if not impossible, to demonstrate that the thermometer, Y, shows the exact temperature of the inner surface of the tube. It was subjected to three tests.

1. The instrument being out of the Sun's rays, and X and Y showing the same temperature, it was removed to a place where the atmospheric temperature was 10° lower. Both thermometers descended and showed a difference equal to about one-tenth the amount they had to fall to arrive at the atmospheric temperature, Y being so much in advance of X.

2. A bat's-wing flame of gas was brought within 3 inches fronting the middle of the tube; both X and Y rose together, keeping pace exactly.

3. While taking observations, the heat absorbed by X from the Sun, and again emitted from it and transferred to the tube, gradually raised its temperature until a maximum was obtained. Now, comparing X and Y while both are rising, and after having obtained their maximum, a difference of 0°·3 was remarked, and this difference, no doubt, affected isolated observations when this maximum was not attained, in consequence of interruption by clouds passing, when it was usual to heat the solar thermometer artificially to near the stationary point, in order to save time; the great inconvenience of the apparatus in this climate being the slowness with which *r* obtained its final value. An arrangement with a differential air-thermometer would, no doubt, be preferable in this respect, but the absolute value of the degrees indicated does not seem capable of being exactly determined.

The thermometers were carefully graduated and compared by myself, and the divisions between two fixed points, 60° and 100° (which included all the observed temperatures), were drawn as nearly equal as a good ivory scale and magnifying lens would admit. The length of a degree on the scale of Y was about ·05 inch, and upon X ·067 inch; with practised eye it was easy to read off the temperature to $\frac{1}{20}$th of a degree with lens; but such accuracy was unattainable for other reasons, and chiefly a sensible difference was caused by the varying amount of the stem that was under the influence of the Sun's rays as it moved.

The observations taken on the morning of the 21st August continuously, during $2\frac{1}{2}$ hours of uninterrupted sunshine, were graphically equalised, the curve drawn and ordinates measured off at every 20 minutes. This was the only opportunity that occurred of continuous observation between such favourable

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limits of altitude as to indicate the direction of a line with some precision.

In the table of observations, given in the Appendix, the date and apparent time are given in the first column. The time-piece was regulated daily by the one o'clock signal-gun. The second column contains the values of r, the observed difference between thermometers X and Y. The film of talc that was interposed between the Sun and X was found to reflect $\frac{9}{50}$th of the incident rays. This ratio was determined by observations taken with the film off during calm weather. The value of r without the film to r with the film on, was as 1·18 to 1·00; the sun's power not sensibly varying during the interval. This proportion was maintained at low values of r, and even when the source of radiation was a gas-flame. The third column contains the observed values of r increased in this ratio. The fourth column contains the corrections required to reduce the values in the third to a vacuum. The correction is taken from a scale that was constructed by means of an empirical formula derived from observations on the cooling of X, as detailed in the Appendix. The fifth column is the final value of r as it would appear in a naked vacuum, that is, a vacuum without any interposed transparent solid between the Sun and the bulb of the thermometer. The numbers in this column represent the quantity of heat-force supplied from the Sun to the bulb of the thermometer in a constant element of time, or the quantity that emanates from the bulb in a unit of time.

The experiments on the cooling of the thermometer, X, in a vacuum, show that from $r = 30^\circ$ to $r = 15^\circ$, the time of cooling was 294 beats of a time-piece, of which $77\frac{1}{2}$ were equal to 60 seconds; also from $r = 15^\circ$ to $r = 7\frac{1}{2}^\circ$, the time was the same, and generally from $r = 2\,m$ to $r = m$ the elapsed time is the constant 294, which thus represents the logarithm of 2 in the logarithmic curve, of which the ordinates are r and the abscissæ the time of cooling, t. The equation of the curve being $c \log \frac{r_1}{r_0} = t_1 - t_0$, in which $c \log 2 = 294$, or to reduce to seconds, $c \log 2 = 294 \times \frac{60}{77\frac{1}{2}}$, and $c = 756 \cdot 1$. Let $t_1 - t_0 = \delta t$, then $\log \frac{r_1}{r_0} = \frac{1}{\mu} \cdot \frac{\delta r}{r}$, in which $\mu =$ hyp. log of 10; hence

$$\frac{c}{\mu} \cdot \frac{\delta r}{r} = \delta t \qquad \left[\log \frac{c}{\mu} = 2 \cdot 51636\right].$$

From this we may compute the quantity of heat supplied to a unit of surface by the Sun in a unit of time corresponding to any value of r. As an example, suppose $r = 10^\circ$ and $\delta t = 1$ second, then $\delta r = \frac{10\,\mu}{c} = 0^\circ \cdot 030453$, or $3^\circ \cdot 0453$ in 100 seconds, is the rate at which the Sun communicates heat to a thermometer, whose bulb is a sphere 0·42 inch in diameter, when $r = 10^\circ$.

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Suppose the glass of the bulb to be $\frac{1}{50}$th of an inch thick, there would be ·0108 cubic inch glass and ·0287 cubic inch mercury heated 3°·045 in 100 seconds. If $r = 20°$, the same heating would take place in 50 seconds, and so on.

To reduce this to thickness of ice melted in 1 minute, we have

Specific heat of mercury ·033 and of glass ·177.

Specific gravity of mercury 13·5 and of glass 2·9.

·0108 cubic inch glass equal in weight to ·0313 cubic inch water.

·0287 cubic inch mercury equal in weight to ·387 cubic inch water.

·0108 cubic inch glass raised 3°·045 takes as much heat as ·0313 cubic inch water raised 0°·54.

·0287 cubic inch mercury raised 3°·045 takes as much heat as ·387 cubic inch water raised 0°·101.

·0313 cubic inch water raised 0°·54 takes as much heat as is required to raise 1 cubic inch 0°·0169.

·387 cubic inch water raised 0°·101 takes as much heat as is required to raise 1 cubic inch 0°·0391.

The entire bulb of the thermometer thus raised 3°·045 is thus equal to 1 cub. in. of water raised ·0169 + ·0391 = 0°·056.

Now, the transverse section of bulb is 0·138 square inch; and since specific gravity of ice is 0·93, and it requires 140° to melt ice, we have $140 \times 0·738 \times 0·93 \times x = 3·045$; hence $x = 0·00312$ inch, the thickness of ice melted by the Sun in 100 seconds, when $r = 10°$. This is equivalent to 0·001872 inch in 1 minute. With $r = 20°$ the thickness would be double this amount, and so on. Thus the presumed extra atmospheric value of r being 67° gives 0·0124 inch thickness melted per minute.

From June to December the amount may be expected to vary $\frac{1}{15}$th, corresponding to alteration of Sun's distance. In Herschel's *Meteorology* the probable thickness is stated to be ·0109 inch.

If the law indicated by straight lines on the chart is true it would require extremely accurate observations to give the extra atmospheric constant of solar radiation with precision. From a single observation made in Bombay some years ago I am disposed to believe it may exceed 67° considerably.

The mode of approaching the law of absorption is as follows:—Project the values of r as ordinates to the secants of zenith distances as abscissæ. The resulting curve is evidently hyperbolic in character. If it is the conic hyperbola, the reciprocals of the ordinates laid off to the same abscissæ should range in a straight line. The obvious plan is, therefore, to lay off the reciprocals of r in this way, and see how far their range agrees with the straight, and, if it differs, the character of the

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divergence might lead us to the true function that expresses the natural law, if it was not very complicated, and if the condition of the atmosphere did not vary so rapidly as to obscure it.

The observations, though taken under unfavourable conditions, favour the simple hyperbola.

It will be remarked, on inspecting the chart, that the value of r at the same altitude of the Sun diminishes with the declination as the season advances. If continuous observations were possible for a few hours each day, when the altitude of the sun ranged between 15° and 45°, we might expect to see the projection of the equalised observations range each day in a different line; but these lines ought all to converge on nearly the same point in the ordinate at the zero of the secant scale, if the law holds good. The lines A C, A D, A E, will exemplify this.

Let N, fig. 3, be the position of the observer, Z his zenith, and NS the direction of the sun. Draw parallel lines *a b*, *c d*, &c.; now *a c* : *b d* : : rad.: sec. sun's zenith distance, so that the thickness of each stratum varies as the secant; and if the physical condition of the stratum did not alter between two observations, we may take the secant as the representative of the collective thickness of the absorbing medium traversed by the Sun's rays, except at such low altitudes when the curvature of the earth as well as refraction may be expected to introduce uncertainty. The minimum value of the secant is radius, but we may imagine the Sun's rays to pass through a similarly constituted atmosphere in which the thickness of the same layers proportionally diminishes from unity or radius to zero. The reciprocal of r diminished for values below radius at the same rate as for values above radius, attains at zero the extra atmospheric limit which, in all climates and seasons, ought to be determined by the inverse square of our planet's distance from the Sun in its orbit, and should not vary beyond $\frac{1}{30}$th of its mean value.

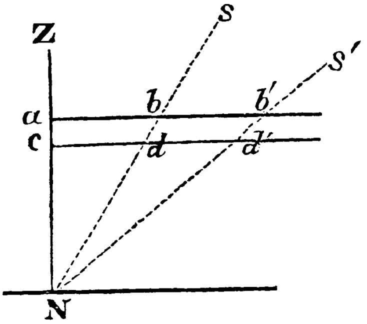

Fig. 3.

Le m_0, m_1, be the secants at which the radiation is r_0, r_1, we have, according to the projection $\dfrac{m_0 - m_1}{\frac{1}{r_0} - \frac{1}{r_1}} = k$, a constant quantity, so long as the physical condition of the atmosphere remains constant; and to find R, the extra atmospheric value of r, we have

$$m_0 - m_1 : \frac{1}{r_0} - \frac{1}{r_1} :: m_1 : \frac{1}{r_1} - \frac{1}{R}.$$

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Hence $m = \frac{k}{r} - \frac{k}{\mathrm{R}}$ and $\frac{1}{\mathrm{R}} = \frac{1}{r} - \frac{m}{k}$. Since R is constant, we may put $\frac{k}{\mathrm{R}} = e$, so that $r = \frac{k}{m + e}$. This expresses the presumed law of absorption or interception.

The essential nature of this law is seen by studying the proportionate differential of r,

$$- \delta r = \frac{k \, \delta m}{(m + e)^2} \text{ and } \frac{- \delta r}{r} = \frac{\delta m}{m + e} = \delta m \frac{r}{k}; \quad \text{hence } \frac{- \delta r}{\delta m} \propto r^2.$$

Thus the Sun's rays, in passing through a constant element of the thickness of the atmospheric medium, loses a proportionate amount of its power that is not constant, but that diminishes in the simple ratio of that power.

As an example, suppose with $r = 30°$, the value diminishes 1° in passing through 1 mile, it would only lose $\frac{1}{4}°$ in passing through the same mile if $r = 15°$, and $\frac{1}{900}$th of a degree if $r = 1°$. We might thus expect, when the atmosphere is clear, it does not intercept any sensible proportion of the heat radiated from the earth's surface into space.

Compare the value of r with one Sun and with two; the supply from each, supposed equal, doubles the value of r, which, measured at the extremities of the mile nearest and furthest from them, shows that for the same element of the thickness of the medium the proportionate decrement of r is constant. Let α represent the angular space occupied by the Sun's disk, and t the potential temperature of its radiating surface, then $t \alpha$ represents the supply of heat by radiation from it upon a unit surface, and is measured by r, so that if αt becomes $2 \alpha t$, r becomes $2 r$. Now, the factor 2 may have reference to α, the magnitude of the Sun's disk, or it may have reference to t, its temperature. The fluctuating value of r from change of altitude or climate represents a fluctuating potential value of αt, but α being constant, the change is similar to what would take place above the atmosphere by a change in t alone. At different parts of the earth's orbit the value of α changes, so that with t constant and α variable the proportionate absorption represented by $\frac{\delta r}{r \, \delta m}$ is a constant quantity, but so far as r depends on t the value $\frac{\delta r}{r \, \delta m}$ increases with t, and the causal relation may be expressed as follows:—

The heat-pulse travels, carrying with it an intensity that it borrows from the temperature of its source, and encounters a deflecting or absorbing power in passing through a constant element of the atmospheric medium that is exactly proportional to that intensity.

It would be simpler if the resistance was uniform;—if the proportion of force absorbed was constant; but the observations do not admit of the possibility of this. The curve traced out

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by the co-ordinates, *r* and secant zenith distance, would in that case be no longer the conic hyperbola, but the logarithmic curve.

At 6 o'clock in the evening of the 31st July, while making an observation, an extensive shower of thin rain took place overhead and westward towards the Sun, without sensibly obscuring its light or affecting its image when examined through a telescope. The value of *r* descended immediately from 15° to 13°. The single observation I took in India, compared with those taken at the same altitude in this country, indicates that the value of *r* is there fully double what it is here, while the quantity of vapour held in suspension estimated from the dew-point is certainly greater. It would seem probable, therefore, that the absorbing power of the atmosphere depends on the watery particles contained in it, not upon the aqueous vapour dissolved in it.

[The Appendix, received simultaneously with the foregoing, will appear in the following number of the *Notices*.—Ed.]

26 *Royal Crescent, Edinburgh, Nov.* 25, 1861.

Explanation of the figures.

Fig. 1. T, U, B, E, is a square tube of brass, mounted with motion in altitude upon an upright, R, fixed into a round slab of lead. The inner surface of this tube is blackened, and at each end, at *l* and *c*, a film of transparent talc was stuck on to prevent the wind from moving the air within the tube.

H, D, D, H, a double screen made of cardboard and cork, coated on both sides with tin-foil, and fitted to slip on the extremity of the tube presented towards the sun.

m, the hole in centre of screen, about $\frac{1}{10}$th inch greater diameter than the bulb of the solar thermometer, X.

The talc film, *l*, was also coated with tin-foil except the central circle.

X, the thermometer in sun with spherical bulb fixed in a cork that fitted the hole, L, L, in top of brass tube.

Y, the thermometer in the shade fixed in the hole, N, N, with cork and soft wax as shown.

Z, a thermometer applied to outer surface of tube.

Fig. 2 is a transverse section of vacuum bath, employed to ascertain the rate of cooling of the solar thermometer, X, in air and in vacuo.

It consists of a cylindrical vessel of brass, coated internally with lamp-black; the lid, L, is ground to the upper edge of the cylinder, and in its centre is a stuffing box, S, with Indian-rubber collar, through which the stem of the thermometer is passed, as shown in the figure; C, is a stop-cock, upon which, N, the nozzle of a flexible tube communicating with an air-pump, is ground air-tight. H is a wooden handle for removing the apparatus to and from the bath without touching the metal.